**Solution set 1**

**Example .no.1 The set of all n-tuples with entities from a field F denoted by fn.This set is a vector space over f with the operation of co-ordinates wise addition and scalar multiplication,that is if**

**U =(a1,a2,.....,an)ϵ Fn,v = (b1,b2,.....,bn)ϵFn  and cϵ f then**

**U+v = (a1 + b1 , a2 + b2 , ........,an + bn) and cu = (ca1 , ca2, ca3 ,....,can)**

Solution: Vs8: **Distributive for scalar addition.**

(a +b)u = au + bu

L.H.s = (a+b)(a1 + a2 + ....+an)

=(a+b)a1 +(a+b)a2 + ......+(a+b)an

= aa1 + ba1 + aa2 + ba2 + .....+ aan+ ban

= aa1 + aa2 + ....+aan + ba1 + ba2 + .....+ban

= a(a1 + a2 +.....an) + b(a1 + a2 + a3+......+an)

= a**u** + b**u**

**Example 2 .The set of m×n matrices with the entities from field F is a vector space,which we denoted by Mm×n (F) with the operation of matrices addition and scalar multiplication (CA)ij = m CAij for 1 ≤ i ≤ m and 1 ≤ j ≤ n.**

Solution: Let us supppse

A = and

B=

Are two m×n matrices ϵ Mm×n(F)

Vs1: A+B = +

= +

= B + A

Vs2: We have (A+B)+C = A+(B+C) where C =

Vs3:There exsit a element Z = Mm×n(F) such that A + Z = A

A + Z = +

Which implies = such that A + 0 = A

Vs4: A = ϵMm×n(F) threre exsist -A =

Such that A + (-A) = +

= Hence A + (-A) = 0

Vs5: × = I ×A = A

Vs6:Let x,y ϵ(F) s.t xy(A) = xy = x = x(yA)

Vs7: x(A + B) = x +

= x

=x+x

= xA + xB

Vs8: for all x,yϵF (x+y) = +

= xA + yB .Hence all the properties of vector space are satisfy hence the set of all m×n matrices with entities from field f is a vector space.

**Excercise.**

**1.Does every vector space contain a zero vector? Justify.Let s = {0,1} and f = R in f(S,R) .show that f = g and f +g = h where f(t) = 2t + 1 ,g(t) = 1 +4t-2t2,and h(t) = 5t+ 1.**

Solution:Yes every vector space contaion a Zero vector .Zero vector is a identity under addition.To be vector space it is necessary to satisfy identity property.

Second part: Here s = { 0,1} and f = R in F (S,R) also, f(t) = 2t + 1,g(t) = 1 + 4t – 2t2 and h(t) = 5t + 1 .

Take t = 0 f(0) = 2.0 + 1 = 1

g(0) = 1 + 4.0 – 2.(0)2 = 1

h(0) = 50+1 = 2 hence f(0) = g(0) so f(t) = g(t)

Take t = 1 f(1) = 2.1 + 1 = 3

g(1) = 1 + 4.1 – 2.(1)2= 3

h(1) = 51+ 1 = 6 hence f(1) + g(1) = h(1) so f(t) +g(t) = h(t) verified.

**2.May a vector space have more than one zero vector?justify.Let v = {(a1,a2):a1,a2ϵR}.for (a1,a2),(b1,b2) = (a1+2b1,a2+3b2) and c(a1,a2) = (ca1,a2).Is v a vector space over f with these operations?justify your answer.**

Solution: Let a vector space have two zeros say 0 and 0’ .From vs3 0 is the identity under addition .hence for all x ϵV

x + 0 = 0+x = x............(\*)

also from vs3 0’ is identity under addition.

X + 0’ = 0’ + x = x..........(\*\*)

From (\*) and (\*\*)

x + 0 = x + 0’  = x .Hence there is unique zeros in vector space.

Second part

Here given v = {(a1,a2):a1,a2ϵR} for (a1,a2),(b1,b2)ϵV and cϵR define (a1,a2)+(b1,b2) = (a1+2b1,a2+3b2) and c(a1,a2) (ca1,a2)

Vs1: for (a1,a2),(b1,b2) ϵV (a1,a2)+(b1,b2) = (a1+2a2,b1+3b2) = (a2+2a1,b2+3b1) = (b1,b2) + (a1,a2)

Vs2: for (a1,a2),(b1,b2)ϵV and (c1,c2) ϵV

[(a1,a2)+(b1,b2)]+(c1,c2) =[ a1+2b1,a2+3b2] +(c1,c2) = (a1+2b1+c1,a2+3b2+3b3)= (a1,a2) + [ b1+2c1,b2+3c2] = (a1,b2) + [(b1,b2)+(c1,c2)]

Vs3:if there exsist a element z = (d1,d2)ϵV such that (a1,a2) +(d1,d2) = (a1,a2)

(a1+2d1,a2+3d2) = (a1,a2)

a 1 +2d1 = a1 =>d1 = 0

a2 +3d2 = a2=> d2 = 0 This show that (a1,a2)+(0,0) = (a1,a2)

vs4:Let for (a1,b1) there exsist (-a1,-a2) ϵV such that (a1,a2) + (-a1,-a2) = [a1+(-2a1),a2+(-3a2)] = (-a1,-2a2) ≠ (0,0)Hence vs4 does not satisfy.

Vs5: 1 ϵ F 1.(a1,b1) = (1.a1,b1) = (a1,b1)

Vs6: for all x,yϵF xy(a1,b1) = x(ya1,b) = x[y(a1,b1)]

Vs7: for all x,yϵF (x+y) (a1,b1) = [(x+y)a1,b1] = (xa1+ya1,b1)

also x(a1,b1) +y(a1,b2) = (xa1, b1) + (ya1,b1) = (xa1+ya1,b1+b2) = (xa1+ya1,2b1)

(x+y)(a1,b1)≠ x(a1,b1) +y(a1,b1) hence vs7 does not satisfy.

Vs8: for x ϵF x[(a1,b1)+(a2,b2)] = x(a1,b1) + x(a2,b2) = (xa1,b1)+(xa2,b2) All the condition of vector space does not satisfy hence it is not vector space.

**3.Does ax = bx imply a = b in any vector space? Justify.let v = ((a1,a2):a1,a2ϵf} where f is field.Define addition of elements of v coordinate wise, and foe c ϵf and (a1,a2)ϵV .Define c(a1,a2) = (a1,0) .Is vector space over F with these operations? Justify your answer by taking all condition of vector space.**

Solution: In any vector space ax = bx implies that a = b false it is only true if we take x = 0 i.e ; ax = bx x(a – b) = 0 either x = 0 or ( a-b) = 0 take x = 0 and any a≠b for x.

Let us suppose (a1,a2),(b1,b2)ϵV then

Vs1: (a1,a2)+(b1,b2) = (a1+b1,a2+b2) = (b1+a1,b2+a2) = (b1,b2)+(a1,a2)

Vs2: [(a1,a2)+(b1,b2)]+(c1,c2) = (a1,a2)+[(b1,b2)+(c1,c2)] for (c1,c2)ϵf

Vs3:There exsit (d1,d2) ϵV s.t(a1,a2)+(d1,d2) = (a1,a2)

Or, (a1,a2) + (0,0) = (a1,a2)

* (d1,d2) = (0,0)

Vs4:for (a1,a2)ϵ V threre exsist (-a1,-a2) ϵV

s.t (a1,a2) + (-a1,-a2) = (0,0)

vs5:for 1ϵF 1.(a1,a2) = (1.a1,0) = (a1,0) ≠ (a1,a2) hence vs5 does not satisfy.

Vs6:for x,y ϵ F xy(a1,a2) = x(ya1,0)

Vs7:for x,yϵ F (x+y) (a1,a2)

= [(x+y)a1,0]

= [xa1+ya1,0]

=(xa1,0)+(ya1,0)

Vs8: for x ϵf x[(a1,a2)+(b1,b2)]

= x[a1+b1,a2+b2]

=[x(a1+b1),0]

=(xa1,0)+(xb1,0)

= x(a1,a2) + x(b1,b2)

Vs5 does not satisfy it is not vector space.

**4.Does ax = bx imply x = y in any vector space?justify.let v denote the set of ordered pairs of real numbers.if (a1,a2) and(b1,b2) are element of v and cϵR ,define (a1 , a2) + (b1 ,b2) = (a1b1,a2+b2) and c(a1,a2) = (a1,ca2) .is v a vector space over R with these operations? Justify your answer by showing all conditions of vector space.**

Solution: In any vector space ax = ay implies that x = y is false.It is only true if we take a = 0

i.e; ax = ay a(x – y) = 0 ϵ V

either a = 0 ,or ( x – y) = 0 if we take a = 0 and any x ≠ y for a.

Second part

Here v denote the set of ordered pairs of real numbers.if (a1,a2) and(b1,b2) are element of v and cϵR ,define (a1 , a2) + (b1 ,b2) = (a1b1,a2+b2) and c(a1,a2) = (a1,ca2).

Vs1: (a1,a2) + (b1 , b2) = (a1b1,a2+b2) = (b1a1 , b2+a2) = (b1,b2) + (a1,a2)

Vs2: for [(a1,a2) +(b1,b2)]+(c1,c2) = (a1b1, a2+b2) + (c1,c2) = (a1b1c1 , a2+b2+c2) = (a1,a2) +(b1c1 , b2 + c2) = (a1,a2) + [(b1,b2) + (c1 , c2)]

Vs3: Threre exsist (d1,d2) ϵV s.t (a1,a2) + (d1,d2) = (a1,a2)

Or, (a1b1 , a2+b2 ) = (a1 , a2) => a1b1 = a1 => b1 = 1

Also, a2 + b2 = a2 => b2 = 0 hence (d1,d2) = (1,0) so (a1,a2) + (d1,d2) ≠ (a1 , a2) vs3 does not satisfy.

Vs4: for (a1 ,a2) there exsist (-a1,-a2) ϵ V s.t (a1,a2) +(-a1 , -a2) = [a1(-a1) , b1 + (-b1) ] = (-a12, 0)≠0 hence vs4 also not satisfy.

Vs5:for 1ϵ R 1.(a1,a2) = (a1, 1.a2) = (a1,a2)

Vs6: for x, yϵR xy(a1,a2) =x (a1,ya2) = x[y(a1,a2)

Vs7: for x,y ϵ R (x + y) [(a1,a2) +(b1 , b2)] = (x+y)(a1,a2) +(x+y) (b1,b2)

L.H.S =(x+y)(a1b1, a2+b2) = [a1b1 , (x+y)(a2 + b2)] =[a1b1 ,(x +y) a2 + (x+y) b2]

R.H.S = [ a1 , (x+y)a2] + [b1 ,(x+y)b2] = [a1b1 , (x+y)a2+(x+y) b2] L.H.S = R.H.S

Vs8: for xϵR x[(a1,a2) +(b1,b2)] = x(a1,a2) + y(b1,b2)

L.H.S = x[ a1b1 ,a2+b2] = [a1b1 , x(a2+b2)] = [a1b1 , xa2 + xb2]

R.H.S = (a1,xa2) + (b1 , xb2) = (a1b1 , xa2 +xb2) L.H.S = R.H.S Here vs3 and vs4 are not satisfy hence it is not vector space.

**5. a. Is the empty set a subspace of every vector space? Justify and let s = {(a,b,(a+b):a,bϵR) .Is a subspace of R3 under the usual operation.**

**b. Let I = (-a , a) a >0 be an open interval in R let v = Rt the space of all real valued function define on I. Let ve ={ fϵv : f(-x) = f(x)} for all x ϵ I the set of all even function on I. And let vo = {fϵv :f(-x) = -f(x) for all xϵ I} ,the set of all odd function on I.Then show that v = ve direct sum v0.**

Solution:

The answer is no.The empty set is empty in the sence that it does not contain any elements.Thus a zero vector is not member of the empty set.without zero we can not say that it is subspace of vector space.

Here s = {a,b,(a+b): a,b ϵR} for a there exsist –a vss1: a + (-a) = 0 ϵs

Vss2: a+b ϵs

Vss3: for c ϵR s.t caϵ s hence three condition of vector sub space are satisfy so, s is vector subspace.

For solution of b.

Here given two function are define by ve = {f ϵv : f(-x) = f(x) for all xϵI} and v0 = {f ϵv :f(-x) = -f(x) for all xϵ I}

First we show that ve and vo are subspace of vector space.

For even: Here **ve ={ fϵv : f(-x) = f(x) for all x ϵ I}**

**Vss1: for all x ϵ I there exsist (-x) ϵ I s.t f(x) +f(-x) = f(x) – f(x) = 0 ϵI**

**VSS2: for all xϵ I x + x = 2x ϵ I**

**VSS3: for all cϵ R => cx ϵI**

**For odd:Here vo = {fϵv :f(-x) = -f(x) for all xϵ I}**

**Vss1: for all x ϵ I there exsist (-x) ϵ I s.t f(x) +f(-x) = f(x) – f(x) = 0 ϵI**

**Vss2: for all x ϵI x + x ϵI sum of odd function is also odd**

**Vss3: for all cϵ R => cx ϵI**

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F(x) = g(x) + h(x), xϵI , where g(x) = ϵve { cos function is even cos(x) = }

h(x) = v0 {sin function is odd sin(x) =}

it follows that v = ve + v0 .also if fϵ veᴒv0 then f ϵ ve and fϵvo ,then f(x) = f(-x) and f(x) = -f(x) and so for all xϵI , 2 f(x) = 0 that is

f = 0 hence v = ve direct sum vo.